DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD 402 103

Summer End Semester Examination –2022

Branch: B. Tech. (Common to all) **Semester: II**

Subject with Subject Code: Engineering Mathematics – II (BTBS 201) **Marks: 60**

Date: 17/08/2022 **Time: 3.45 Hrs.**

Instructions to the Students

1. Illustrate your answers with neat sketches, diagrams, etc., wherever necessary.
2. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

Q. 1

(a) If the sum and product of two complex numbers are real, show that those two numbers must be either real or conjugate. [4 *Marks*]

(b) Solve the equation $x^6 - i = 0$. [4 *Marks*]

(c) If $\tan(A + iB) = x + iy$, prove that

(i) $\tan 2A = \frac{2x}{1-x^2-y^2}$ (ii) $\tanh 2B = \frac{2y}{1+x^2+y^2}$. [4 *Marks*]

Q. 2

(a) Solve: $\cos^2 x \frac{dy}{dx} + y = \tan x$. [4 *Marks*]

(b) Solve: $(x^2 + y^2)dx - xy\, dy = 0$. [4 *Marks*]

(c) Solve: $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$. [4 *Marks*]

Q. 3 Solve any THREE:

(a) Solve $(D^6 - D^4)y = x^2$. [4 *Marks*]

(b) Solve $(D^2 - 2D + 1)y = x\, e^x \cos x$. [4 *Marks*]

(c) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$. [4 *Marks*]

$PI = -y_1 \int \frac{y_2 X}{W} dx + y_2 \int \frac{y_1 X}{W} dx = -x\cos x + \sin x \log \sin x$

(d) Solve: $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)$. [4 *Marks*]

Q. 4 Solve any TWO:

(a) Find the Fourier series of the function $f(x) = x$ in the interval $(0, 2\pi)$. [6 M

(b) Find the Fourier series expansion for the function $f(x) = x - x^2$ in $-1 < x < 1$. [6 M

(c) Expand the function $f(x) = \pi x - x^2$ in a half-range sine series in the interval $(0, \pi)$. [6 M

Q. 5 Solve any THREE

(a) Find $\nabla \cdot \vec{F}$, where $\vec{F} = \left(\frac{x}{r}\right)\hat{\imath} + \left(\frac{y}{r}\right)\hat{\jmath} + \left(\frac{z}{r}\right)\hat{k}$. [4 Mar

(b) Find curl $\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. [4 Mar

(c) If $\vec{r}$ is a position vector with $r = |\vec{r}|$, show that

$\nabla^2 r^n = n(n+1)r^{n-2}$. [4 Mar

(d) Verify the Green's theorem for $\int_C \{(xy + y^2)dx + x^2 dy\}$

where C is bounded by $y = x$ and $y = x^2$. [4 Mark

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DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End – Semester Examination (Supplementary): May 2019

Branch: B. Tech (Common to all) **Semester:** II

Subject with code: Engineering Mathematics – II (MATH 201) **Marks:** 60

Date: 29.05.2019 **Duration:** 03 Hrs.

INSTRUCTION: Attempt any **FIVE** of the following questions. All questions carry equal marks.

Q.1

(a) Find all the values of $(i)^{\frac{1}{4}}$ [4 Marks]

(b) If $sin(\theta + i\phi) = cos\alpha + isin\alpha$, prove that $cos^2\theta = \pm sin\alpha$. [4 Marks]

(c) Prove that $\tan\left[i\, log\frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2-b^2}$. [4 Marks]

Q.2

(a) Solve: $cos^2x\frac{dy}{dx} + y = tanx$. [4 Marks]

(b) Solve: $(x^2 + y^2)dx - (xy)dy = 0$. [4 Marks]

(c) Two particles fall freely, one in a medium whose resistance is equal to k times the velocity and other in a medium whose resistance is equal to k times the square of the velocity. If V_1 and V_2 are their maximum velocities respectively, show that $V_1 = V_2^2$. [4 Marks]

Q.3 Solve any TWO:

(a) Solve: $(D^2 - 3D + 2)y = e^{3x}$. [6 Marks]

(b) Solve: $(D^6 - D^4)y = x^2$. [6 Marks]

(c) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + y = cosecx.$$

[6 Marks]

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Q.4

(a) Find the Fourier series of $f(x) = x^2$ in the interval $(-\pi,\ \pi)$, and hence deduce that

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

[6 *Marks*]

(b) If $f(x) = 2x - x^2$ in $0 \le x \le 2$, show that $f(x) = \frac{2}{3} - \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2} \cos n\pi x$.

[6 *Marks*]

Q.5

(a) The necessary and sufficient condition for vector $\vec{F}(t)$ to have constant magnitude is

$\vec{F}(t) \cdot \frac{d\vec{F}(t)}{dt} = 0$.

[6 *Marks*]

(b) Show that the acceleration of the point moving along the curve with uniform speed is $\varrho\left(\frac{d\psi}{dt}\right)^2$ along the normal.

[6 *Marks*]

Q.6

(a) Find $\nabla \cdot \vec{F}$, where $\vec{F} = \nabla\,(x^3 + y^3 + z^3 - 3xyz)$.

[4 *Marks*]

(b) If $\vec{r}$ is a position vector with $r = |\vec{r}|$, show that

$\nabla \cdot (r^n\, \vec{r}) = (n+3)r^n$.

[4 *Marks*]

(c) Show that $\iiint_v \frac{dv}{r^2} = \iint_s \frac{\vec{r}\cdot\hat{n}}{r^2}\, ds$.

[4 *Marks*]

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DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination – Summer 2019

Course: B. Tech in All Branches **Sem: II**

Subject Name: Engineering Mathematics II **Subject Code: BTMA201**

Max Marks:60 **Date:13/05/2019** **Duration: 3 Hr.**

Instructions to the Students:

1. *Solve ANY FIVE questions out of the following.*
2. *Use of non-programmable scientific calculators is allowed.*
3. *Assume suitable data wherever necessary and mention it clearly.*
4. *Figures to the right indicate full marks.*

Q. 1 Solve Any Three of the following. 12

A) If $\arg(z+1) = \frac{\pi}{6}$ and $\arg(z-1) = \frac{2\pi}{3}$, find z.

B) If $\alpha = 1+i$, $\beta = 1-i$ and $\cot\theta = x+1$, prove that

$$(x+\alpha)^n + (x+\beta)^n = (\alpha-\beta)\sin(n\theta)\, cosec^n(\theta).$$

C) Show that all the roots of $(x+1)^6 + (x-1)^6 = 0$ are given by $-i\cot\left(\frac{(2k+1)\pi}{12}\right)$, k=0,1,2,3,4,5.

D) If $\tan(\theta + i\emptyset) = \cos\alpha + i\sin\alpha$, prove that

I. $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$.

II. $\emptyset = \frac{1}{2}\log_e \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$.

Q.2 Solve Any Three of the following. 12

A) Solve $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$.

B) Solve $ydx - xdy + \log x\, dx = 0$.

C) Find the orthogonal trajectories of $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ is a parameter.

D) A constant electromotive force E volts is applied to a circuit containing a constant resistance R ohm in series and a constant inductance L henries. If the initial current is zero , show that the current builds up to half its theoretical maximum in (L log2)/R sec.

P.T.O.

Q.3) Solve Any Three of the following. 12

A) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$.

B) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = e^{2x} + \sin x + x$.

C) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$.

D) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$.

Q.4 Solve Any Two of the following. 12

A) Find the Fourier series for $f(x) = \sqrt{1 - \cos x}$ in the range $(0, 2\pi)$. Prove that $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{4n^2-1}$.

B) Obtain the Fourier series for $f(x)$ given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi. \end{cases}$$ Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots = \frac{\pi^2}{8}$.

C) If $f(x) = 2x - x^2$ in $0 \le x \le 2$, show that $f(x) = \frac{2}{3} - \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2} \cos(n\pi x)$.

Q.5 Solve Any Three of the following. 12

A) Find the directional derivatives of $\emptyset = e^{2x} \cos yz$ at (0,0,0) in the direction of the tangent to the curve $x = a \sin t, y = a \cos t, z = at$ at $t = \frac{\pi}{4}$.

B) Find the cosine of the angle between the normals to the surfaces $x^2y + z = 3$ and $x \log z - y^2 = 4$ at the point of intersection $p(-1,2,1)$.

C) Find $curl\ \vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$.

D) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, find $\vec{r}.\nabla\emptyset$ for $\emptyset = x^3 + y^3 + z^3 - 3xyz$.

Q. 6 Solve Any Two of the following. 12

A) Evaluate $\oint_C \vec{F}.d\vec{r}$ where C is the square formed by the lines $y = \pm 1$ and $x = \pm 1$, and $\vec{F} = (x^2 + xy)\hat{\imath} + (x^2 + y^2)\hat{\jmath}$.

B) Verify the Green's theorem for $\int_C \{(xy + y^2)dx + x^2dy\}$ where C is bounded by $y = x$ and $dy = x^2$.

C) Evaluate $\iint_S \{2x^2ydydz - y^2dzdx + 4xz^2dxdy\}$ over the curved surface of the cylinder $y^2 + z^2 = 9$, bounded by $x = 0$ and $x = 2$.

*** End ***

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY,
LONERE – RAIGAD -402 103
Semester Winter Examination – December - 2019

Branch: B. Tech. (Common to all) Semester:- II

Subject with Subject Code:- Engineering Mathematics – II (MATH 201) Marks: 60

Date:- 09/12/2019 Time:- 3 Hr.

Instructions to the Students

1. Attempt any five questions of the following.
2. Illustrate your answers with neat sketches, diagram etc., wherever necessary.
3. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly

Q.1

(a) Find all the values of $(i)^{\frac{1}{4}}$. [4 Marks]

(b) If $\tan(A+iB)=(x+iy)$, prove that

(i) $\tan 2A = \frac{2x}{1-x^2-y^2}$ (ii) $\tan h2B = \frac{2y}{1+x^2+y^2}$. [4 Marks]

(c) Prove that $\log(1+e^{2i\theta}) = \log(2cos\theta) + i\theta$. [4 Marks]

Q.2

(a) Solve: $(x^2-y^2)dx = 2xy\,dy$. [4 Marks]

(b) Solve: $(y+logx)dx - (x)dy = 0$. [4 Marks]

(c) Two particles fall freely, one in a medium whose resistance is equal to k times the velocity and other in a medium whose resistance is equal to k times the square of the velocity. If V_1 and V_2 are their maximum velocities respectively, show that $V_1 = V_2^2$. [4 Marks]

Q.3 Solve any TWO:

(a) Solve: $(D^2-3D+2)y = e^{3x}$. [6 Marks]

(b) Solve: $(D^2-2D+1)y = x\,e^x\,sinx$. [6 Marks]

(c) Solve by the method of variation of parameters

$\frac{d^2y}{dx^2} + y = cosecx$. [6 Marks]

Q.4

(a) Find the Fourier series of $f(x) = x^2$ in the interval $(0\ ,\ 2\pi)$, and hence deduce that [6 Marks]

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

(b) Expand the function $f(x) = \pi x - x^2$ in a half – range sine series in the interval $(0, \pi)$. [6 Marks]

Q.5

(a) The necessary and sufficient condition for vector $\vec{F}(t)$ to have constant magnitude is

$$\vec{F}(t)\,.\frac{d\vec{F}(t)}{dt} = 0\,.$$ [6 Marks]

(b) A point moves in a plane so that its tangential and normal components of acceleration are equal and angular velocity of the tangent is constant and equal to ω. Show that the path is equiangular spiral $\omega s = Ae^{\omega t} + B$, where A and B are the constant.

[6 Marks]

Q.6

(a) Find Curl $\vec{F}$, where $\vec{F} = \nabla\,(x^3 + y^3 + z^3 - 3xyz)$. [4 Marks]

(b) If $\vec{r}$ is a position vector with $r = |\vec{r}|$, show that

$$\nabla \times (r^n\,\vec{r}) = 0.$$ [4 Marks]

(c) Show that $\iiint_v \frac{dv}{r^2} = \iint_s \frac{\vec{r}\,.\,\hat{n}}{r^2}\,ds$. [4 Marks]

****************************Paper End******************************

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE – RAIGAD
Semester Winter Examination – December - 2019

Branch: B. Tech. (Common to all) Semester: II

Subject with Subject Code: Engineering Mathematics – II (BTMA 201) Marks: 60

Date: 09.12.2019 Time. 3 Hrs.

Instructions to the Students

1. Attempt **any five** questions of the following.
2. Illustrate your answers with neat sketches, diagrams, etc., wherever necessary.
3. If some part or parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

Q. 1

(a) If the sum and product of two complex numbers are real, show that those two numbers must be either real or conjugate. [4 *Marks*]

(b) Solve the equation $x^6 - i = 0$. [4 *Marks*]

(c) If $\tan(A + iB) = x + iy$, prove that

(i) $\tan 2A = \frac{2x}{1-x^2-y^2}$ (ii) $\tanh 2B = \frac{2y}{1+x^2+y^2}$. [4 *Marks*]

Q. 2

(a) Solve: $\cos^2 x \frac{dy}{dx} + y = \tan x$. [4 *Marks*]

(b) Solve: $(x^2 + y^2)dx - xy\,dy = 0$. [4 *Marks*]

(c) A body falling from rest is subjected to the force of gravity and an air resistance of $\left(\frac{n^2}{g}\right)$ times square of the velocity. Show that the distance travelled by the body in t seconds is $\frac{g}{n^2} \log \cosh(nt)$. [4 *Marks*]

Q. 3 Solve any THREE:

(a) Solve $(D^6 - D^4)y = x^2$. [4 *Marks*]

(b) Solve $(D^2 - 2D + 1)y = x\, e^x \cos x$. [4 *Marks*]

(c) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$. [4 *Marks*]

(d) Solve: $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$. [4 *Marks*]

Q. 4 Solve any TWO:

(a) Find the Fourier series of the function $f(x) = x$ in the interval $(0, 2\pi)$. [6 *Marks*]

(b) Find the Fourier series expansion for the function $f(x) = x - x^2$ in $-1 < x < 1$. [6 *Marks*]

(c) Expand the function $f(x) = \pi x - x^2$ in a half-range sine series in the interval $(0, \pi)$. [6 *Marks*]

Q. 5 Solve any THREE

(a) Find the value of the constant λ such that the vector field defined by

$\vec{F} = (2x^2y^2 + z^2)\hat{\imath} + (3xy^3 - x^2z)\hat{\jmath} + (\lambda xy^2z + xy)\hat{k}$ is solenoidal. [4 *Marks*]

(b) Find $\nabla \cdot \vec{F}$, where $\vec{F} = \left(\frac{x}{r}\right)\hat{\imath} + \left(\frac{y}{r}\right)\hat{\jmath} + \left(\frac{z}{r}\right)\hat{k}$. [4 *Marks*]

(c) Find **curl** $\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. [4 *Marks*]

(d) If $\vec{r}$ is a position vector with $r = |\vec{r}|$, show that

$\nabla^2 r^n = n(n+1)r^{n-2}$. [4 *Marks*]

Q. 6:

(a) Find the values of the line integral $\int_C \vec{F} \cdot d\vec{r}$ along the path

$y^2 = x$ joining the points $(0, 0)$ and $(1,1)$ provided that $\vec{F} = x^2\hat{\imath} + y^2\hat{\jmath}$. [4 *Marks*]

(b) Verify the Green's theorem for $\int_C \{(xy + y^2)dx + x^2dy\}$

where C is bounded by $y = x$ and $y = x^2$. [4 *Marks*]

(c) Show that $\iiint_v \frac{dv}{r^2} = \iint_s \frac{\vec{r} \cdot \hat{n}}{r^2}\, ds$. [4 *Marks*]

***************************Paper End********************************

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End – Semester Examination (Supplementary): November 2018

Branch: B. Tech (Common to all) **Semester:** II

Subject with code: Engineering Mathematics – II (MATH 201)

Date: 27/11/2018 **Max Marks:** 60 **Duration:** 03 Hrs.

INSTRUCTION: Attempt any **FIVE** of the following questions. All questions carry equal marks.

Q.1 (a) Prove that $cos^6\theta - sin^6\theta = \frac{1}{16}(cos6\theta + 15cos2\theta)$. [6 Marks]

(b) If $an(A + iB) = x + iy$, prove that

(i) $tan2A = \frac{2x}{1-x^2-y^2}$ (ii) $tanh2B = \frac{2y}{1+x^2+y^2}$. [6 Marks]

Q.2 (a) Solve $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$. [6 Marks]

(b) Solve

$$x - xdy + logxdx = 0.$$ [6 Marks]

Q.3 **Solve any TWO:**

(a) Solve $y'' + 4y' + 13y = 18e^{-2x}$ [6 Marks]

(b) Solve $(D^2 + 5D + 4)y = x^2 + 7x + 9$. [6 Marks]

(c) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + y = cosecx.$$ [6 Marks]

Q.4 (a) Find the Fourier series of $f(x) = x^2$ in the interval $(0,\ 2\pi)$ and hence deduce that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$ [6 Marks]

(b) Expand the function $f(x) = \pi x - x^2$ in a half – range sine series in the interval $(0,\ \pi)$.

[6 Marks]

P.T.O.

Q.5 (a) The necessary and sufficient condition for vector $\vec{F}(t)$ to have constant magnitude is

$$\vec{F}(t).\frac{d\vec{F}(t)}{dt} = 0.$$

[6 Marks]

(b) A point moves in a plane so that its tangential and normal components of acceleration are equal and the angular velocity of the tangent is constant and equal to ω. Show that the path is equiangular spiral $\omega s = Ae^{\omega t} + B$, where A & B are constants. [6 Marks]

Q.6 Solve any TWO:

(a) Find curl$\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. [6 Marks]

(b) If $\vec{r}$ is a position vector with $r = |\vec{r}|$, show that

$$\nabla.(r^n\vec{r}) = (n+3)r^n$$

[6 Marks]

(c) Show that $\iiint_v \frac{dv}{r^2} = \iint_s \frac{\vec{r}.\hat{n}}{r^2} ds$ [6 Marks]

Dr. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination: May 2018

Branch:	B.Tech (Common to all)	Semester:	II
Subject with code:	Engineering Mathematics-II (MATH 201)	Marks:	60
Date:	14/05/2018	Time:	03 Hrs.

INSTRUCTION: Attempt any FIVE of the following questions. All questions carry equal marks.

Q.1 Solve any three

(a) If $\arg(z+1)=\frac{\pi}{6}$ and $\arg(z-1)=\frac{2\pi}{3}$ find z [4 Marks]

(b) Solve: $x^7+x^4+x^3+1=0$ [4 Marks]

(c) If $\cos(\theta+i\phi)=Re^{i\alpha}$, show that $\phi=\frac{1}{2}\log\left(\frac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}\right)$ [4 Marks]

(d) Prove that $\tan\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\}=\frac{2ab}{a^2-b^2}$ [4 Marks]

Q.2 Solve any three.

(a) Solve $(4x-6y-1)dx+(3y-2x-2)dy=0$ [4 Marks]

(b) Solve $\frac{dy}{dx}=\frac{y+1}{(y+2)e^y-x}$ [4 Marks]

(c) Solve $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$ [4 Marks]

(d) Determine the charge and current at any time t in a series R-C circuit with $R=10\Omega$, $C=2\times10^{-4}$ F and E = 100V given that q(0) = 0 [4 Marks]

Q.3. Solve any three.

(a) Solve $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=5^x-\log 2$ [4 Marks]

(b) Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+5y=25x^2$ [4 Marks]

(c) Solve $(D^2+2D+1)y=e^{-x}\log x$ by method of variation of parameters. [4 Marks]

(d) Solve $x^2y''-3xy'+5y=x^2\sin(\log x)$ [4 Marks]

Q.4. (a) Obtain the Fourier series expansion of $\sqrt{1-\cos x}$ in the interval $0 \leq x \leq 2\pi$. [6 Marks]

(b) Find the Half-range co-sine series for $f(x) = \begin{cases} \frac{1}{4} - x & 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \frac{1}{2} < x < 1 \end{cases}$ [6 Marks]

Q.5. (a) If a particle describes the curve $r = 2a\cos\theta$ with constant angular speed ω, find the radial and transverse components of velocity and acceleration. [4 Marks]

(b) For the curve $x = t^3 + 1, y = t^2, z = t$, find the magnitude of tangential and normal components of acceleration at $t = 1$. [4 Marks]

(c) If the particle describes the cardioid $r = a(1-\cos\theta)$ under a force to the pole, show that the force is proportional to the inverse of the 4th power of the distance. [4 Marks]

Q.6. (a) Find the directional derivative of $\phi = 5x^2y - 5y^2z + 2.5z^2x$ at the point $p(1,1,1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = z$. [4 Marks]

(b) If $\vec{F} = (ax+3y+4z)\hat{i} + (x-2y+3z)\hat{j} + (3x+2y-z)\hat{k}$ is solenoidal, find the value of 'a'. [4 Marks]

(c) Find the total work done in moving a particle in the force field, given by $\vec{F} = 3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. [4 Marks]

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